

Phase diagram of UCoGe

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The temperature-pressure phase diagram of ferromagnetic superconductor UCoGe includes four phase transitions. They are between the paramagnetic and the ferromagnetic states with the subsequent transition in the superconducting ferromagnetic state and between the normal and the superconducting states after which has to occur the transition to the superconducting ferromagnetic state. Here we have developed the Landau theory description of the phase diagram and established the specific ordering arising at each type of transition. It is shown in particular that the direct phase transition from the nonmagnetic normal to the ferromagnetic superconducting state is inadmissible.

The phase transitions to the ferromagnetic superconducting state are inevitably accompanied by the emergency of screening currents. The corresponding magnetostatics considerations allow to establish the significant difference between the transition from the ferromagnetic to the ferromagnetic superconducting state and the transition from the superconducting to the ferromagnetic superconducting state.

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I. INTRODUCTION

The superconductivity in the uranium ferromagnetic compounds UGe₂ and URhGe discovered more than decade ago [1, 2] and more recently in the related compound UCoGe [3] is still the subject of quite active investigations (see recent experimental [4] and theoretical [5] reviews and references therein). The existence of superconducting state at temperatures far below the Curie temperature and very high upper critical field in these materials do not leave doubts that here we deal with the triplet superconductivity like it is in the superfluid ³He. This is also confirmed by the measurements of the Knight shift on nucleus of ⁵⁹Co which is proved unchanged in the superconducting state [6].

One of many peculiar properties of UCoGe is that the ferromagnetism in this compound is suppressed by pressure whereas the superconductivity arising at small pressures inside of the ferromagnetic state continues to exist at high pressures in the paramagnetic state. The pressure-temperature phase diagram shown in Fig.1, has been established first in the paper [7] and then confirmed in many subsequent studies (see f.i. the last one [8]). The phase transition from the paramagnetic to the ferromagnetic state and the following to it the phase transition to the ferromagnetic superconducting state at low pressures and the phase transition from the normal to the superconducting state at high pressures are firmly registered. While the phase transition from the superconducting to the ferromagnetic superconducting state shown on the Fig.1 by the dashed line is still not confirmed experimentally.

A theoretical phase diagram description has been proposed recently by Cheung and Raghunathan [9]. Making use the numerical calculations applied to the minimal Landau model of neutral ferromagnetic superfluid state with one component order parameter for each spin up-up and spin down-down Cooper pair states they were able to

reproduce the general structure of the UCoGe phase diagram and to predict a first order phase transition near the boundary between the normal phase and the ferromagnetic superconducting phase.

Here I reconsider the same problem making use the analytical calculations applied to the same minimal model for neutral ferromagnetic or paramagnetic superfluid states. The results of Ref.9 were confirmed in respect to the transition from the normal to the ferromagnetic state with following to it transition to ferromagnetic superconducting state. However, it is proven that there is no direct phase transition from the nonmagnetic normal to the ferromagnetic superconducting state. As result, this part of the phase diagram has undergone modifications.

A phase transition of normal metallic to the superconducting state has its own specific properties different from the properties of a phase transition in the neutral Fermi liquid to the superfluid state. So, in the last part of the paper I will discuss the significant difference between the two phase phase transitions, namely, between the phase transition from the ferromagnetic normal state to the ferromagnetic superconducting state and the phase transition from the superconducting state to the ferromagnetic superconducting state. This difference arises due to the essentially different screening of magnetic moment at these two transitions.

II. MODEL

The triplet-pairing superconducting state order parameter is given by the complex spin-vector [10]

$$\mathbf{d}(\mathbf{k}, \mathbf{r}) = \frac{1}{2} [-\Delta^\uparrow(\mathbf{k}, \mathbf{r})(\hat{x} + i\hat{y}) + \Delta^\downarrow(\mathbf{k}, \mathbf{r})(\hat{x} - i\hat{y})] + \Delta^0(\mathbf{k}, \mathbf{r})\hat{z}, \quad (1)$$

where $\Delta_{\uparrow}(\mathbf{k}, \mathbf{r})$, $\Delta_{\downarrow}(\mathbf{r}, \mathbf{k}, \mathbf{r})$, $\Delta_0(\mathbf{k}, \mathbf{r})$ are the amplitudes of spin-up, spin-down and zero-spin of superconducting order parameter depending on the Cooper pair centre of gravity coordinate \mathbf{r} and the momentum \mathbf{k} of pairing electrons. In the tetragonal ferromagnets with easy axis along \hat{z} direction there are only two superconducting states A and B with different critical temperature [11]. The general form of the order parameter for the A-state in a two-band spin-up, spin-down superconducting ferromagnet

$$\begin{aligned}\Delta_A^{\uparrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta_x^{\uparrow}(\mathbf{r}) + i \hat{k}_y \eta_y^{\uparrow}(\mathbf{r}), \\ \Delta_A^{\downarrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta_x^{\downarrow}(\mathbf{r}) + i \hat{k}_y \eta_y^{\downarrow}(\mathbf{r}), \\ \Delta_A^0(\mathbf{k}, \mathbf{r}) &= \hat{k}_z \eta_z^0(\mathbf{r})\end{aligned}\quad (2)$$

depends from the five complex amplitudes η_x^{\uparrow} , η_y^{\uparrow} , η_x^{\downarrow} , η_y^{\downarrow} , η_z^0 , which obey to coupled Ginzburg-Landau equations, derived in the linear approximation in the papers [5, 12]. $\hat{k}_i = k_i/|\mathbf{k}|$, $i = x, y, z$ are the projections of the unit $\hat{\mathbf{k}}$ vector on the coordinate axis.

The order parameter of the paramagnetic superconducting state [5, 13] in a orthorhombic metal looks like the order parameter of superfluid $^3\text{He-B}$ phase [10]

$$\begin{aligned}\Delta^{\uparrow}(\mathbf{k}, \mathbf{r}) &= -\hat{k}_x \eta_x(\mathbf{r}) + i \hat{k}_y \eta_y(\mathbf{r}), \\ \Delta^{\downarrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta_x(\mathbf{r}) + i \hat{k}_y \eta_y(\mathbf{r}), \\ \Delta_A^0(\mathbf{k}, \mathbf{r}) &= \hat{k}_z \eta_z(\mathbf{r}).\end{aligned}\quad (3)$$

To avoid excessive difficulties the authors of [9] considered the minimal model for the superconducting ferromagnetic state with the order parameter

$$\begin{aligned}\Delta^{\uparrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta_{\uparrow}(\mathbf{r}), \\ \Delta^{\downarrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta_{\downarrow}(\mathbf{r}).\end{aligned}\quad (4)$$

Corresponding simplest order parameter for the paramagnetic superconducting state looks like the order parameter for the discovered recently polar state of superfluid ^3He [14]

$$\begin{aligned}\Delta^{\uparrow}(\mathbf{k}, \mathbf{r}) &= -\hat{k}_x \eta(\mathbf{r}), \\ \Delta^{\downarrow}(\mathbf{k}, \mathbf{r}) &= \hat{k}_x \eta(\mathbf{r}).\end{aligned}\quad (5)$$

In neglect of interactions of electron charges with magnetic field created by the magnetization one can write following Ref.9 the gradient independent Landau free energy density as

$$F = \alpha M^2 + \beta M^4 + \alpha_1(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2) + \gamma_1 M(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2) + \gamma_2(\eta_{\uparrow} \eta_{\downarrow}^* + \eta_{\uparrow}^* \eta_{\downarrow}) + B(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)^2 + C(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)^2, \quad (6)$$

where M is the density of magnetic moment component along the easy axis,

$$\alpha = \alpha_0(T - T_c), \quad \alpha_1 = \alpha_{10}(T - T_{sc0}), \quad (7)$$

$T_c(P)$ is the pressure dependent Curie temperature and $T_{sc0}(P)$ is the formal critical temperature of superconducting transition in the single band (say just spin-up) case. The phenomenological treatment does not allow to fix the pressure dependences of these critical temperatures and the other coefficients in Eq.(6). In what follows we shall assume that the pressure dependences of these values qualitatively correspond to the phase diagram with the intersection of the phase transition lines shown in Fig.1.

One can note that the symmetry allows also the following interaction $i\gamma_3 M(\eta_{\uparrow} \eta_{\downarrow}^* - \eta_{\uparrow}^* \eta_{\downarrow})$ between the super-

conducting and magnetic order parameters [13], but the general enough microscopic calculations [5, 12] do not confirm the existence of this term.

In general, the free energy fourth order terms actually have the form different from $B(|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)^2 + C(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)^2$ used in Ref.9. If the normal state Green functions are diagonal in the band indices (in our case they are spin-up and spin-down indices) the Wick's decoupling does not produce any mixing terms between the band order parameters (see f.i. [15]). This case the fourth order terms in respect of the superconducting order parameters are

$$\beta_1(|\eta_{\uparrow}|^4 + |\eta_{\downarrow}|^4) + \tilde{\beta}_1 M(|\eta_{\uparrow}|^4 - |\eta_{\downarrow}|^4). \quad (8)$$

If in the normal state there is the band mixing interaction this leads to emergence of the additional terms

$$\beta_2 |\eta_{\uparrow}|^2 |\eta_{\downarrow}|^2 + \beta_3 [(\eta_{\uparrow} \eta_{\downarrow}^*)^2 + (\eta_{\uparrow}^* \eta_{\downarrow})^2] + \beta_4 (|\eta_{\uparrow}|^2 + |\eta_{\downarrow}|^2)(\eta_{\uparrow} \eta_{\downarrow}^* + \eta_{\uparrow}^* \eta_{\downarrow}) + \tilde{\beta}_4 M(|\eta_{\uparrow}|^2 - |\eta_{\downarrow}|^2)(\eta_{\uparrow} \eta_{\downarrow}^* + \eta_{\uparrow}^* \eta_{\downarrow}). \quad (9)$$

One can show that the additional fourth order terms do not introduce a qualitative modification in the phase dia-

gram. So, we will work with the same free energy density as in Ref.9

$$F = \alpha M^2 + \beta M^4 + \alpha_1(|\eta_\uparrow|^2 + |\eta_\downarrow|^2) + \gamma_1 M(|\eta_\uparrow|^2 - |\eta_\downarrow|^2) + \gamma_2(\eta_\uparrow \eta_\downarrow^* + \eta_\uparrow^* \eta_\downarrow) + \beta_1(|\eta_\uparrow|^4 + |\eta_\downarrow|^4) + \beta_2|\eta_\uparrow|^2|\eta_\downarrow|^2. \quad (10)$$

III. PHASE TRANSITIONS

At low pressures the system first passes from the paramagnetic to the ferromagnetic state and then from the ferromagnetic state to the ferromagnetic superconducting state. We begin with consideration of these phase transitions and then discuss the high pressures transitions from the normal to the superconducting state and from the superconducting state to the ferromagnetic superconducting state.

A. Phase transition from the paramagnetic to the ferromagnetic state

The second order transition from paramagnetic to ferromagnetic state occurs at $T = T_{Curie}(P)$. Below this temperature the magnetic moment acquires the finite value and a superconducting ordering is absent

$$M^2 = (M_0(T))^2 = -\frac{\alpha_0(T - T_c(P))}{2\beta}, \quad \eta_\uparrow = \eta_\downarrow = 0. \quad (11)$$

B. Phase transition from the ferromagnetic state to the superconducting ferromagnetic state

At the subsequent phase transition the superconducting order parameter amplitudes $\eta_\uparrow, \eta_\downarrow$ appear and the magnetic moment acquires a magnitude $M = M_0 + m$. Accepting for certainty that coefficient $\gamma_2 = -|\gamma_2|$ is negative we see from Eq. (10) that the phase difference between the superconducting order parameters is absent

$$\eta_\uparrow = \eta_1 e^{i\varphi}, \quad \eta_\downarrow = \eta_2 e^{i\varphi}. \quad (12)$$

Here, η_1 and η_2 are the modules of the superconducting order parameters. Thus, one can rewrite the free energy density (10) as

$$F = \alpha M^2 + \beta M^4 + \alpha_1(\eta_1^2 + \eta_2^2) + \gamma_1 M(\eta_1^2 - \eta_2^2) - 2|\gamma_2|\eta_1\eta_2 + \beta_1(\eta_1^4 + \eta_2^4) + \beta_2\eta_1^2\eta_2^2. \quad (13)$$

The minimization of the free energy density (13) in respect η_1, η_2 and m yields the equations

$$\alpha_1\eta_1 + \gamma_1(M_0 + m)\eta_1 - |\gamma_2|\eta_2 + 2\beta_1\eta_1^3 + \beta_2\eta_1\eta_2^2 = 0, \quad (14)$$

$$\alpha_1\eta_2 - \gamma_1(M_0 + m)\eta_2 - |\gamma_2|\eta_1 + 2\beta_1\eta_2^3 + \beta_2\eta_1^2\eta_2 = 0, \quad (15)$$

$$2\alpha m + 12\beta M_0^2 m + 12\beta M_0 m^2 + 4\beta m^3 + \gamma_1(\eta_1^2 - \eta_2^2) = 0. \quad (16)$$

Here, we have taken into account that M_0 is the minimum of free energy at $\eta_1 = \eta_2 = 0$ and omitted the fourth order terms. The corresponding linear equations for η_1, η_2

$$(\alpha_1 + \gamma_1 M_0)\eta_1 - |\gamma_2|\eta_2 = 0, \quad (17)$$

$$-|\gamma_2|\eta_1 + (\alpha_1 - \gamma_1 M_0)\eta_2 = 0 \quad (18)$$

are not coupled with linear equation for m . Equating the determinant of this system to zero and taking into account Eq.(11) we obtain the equation

$$T_{sc} = T_{sc0} + \frac{\sqrt{(\gamma_1(M_0(T_{sc}))^2 + \gamma_2^2)}}{\alpha_{10}} \quad (19)$$

for the temperature T_{sc} of transition to the superconducting ferromagnetic state. We shall not write the explicit formula for T_{sc} in view of its cumbersome shape. Let

us only note that according to this equation the pressure decrease of the Curie temperature $T_c(P)$ causes the increase of the superconducting transition temperature $T_{sc}(P)$, although this is not the only reason for the $T_{sc}(P)$ pressure dependence.

The linear equation in respect of m gives

$$m \cong -\frac{\gamma_1(\eta_1^2 - \eta_2^2)}{8\beta M_0^2}. \quad (20)$$

So, m is proved to be of the next order of smallness in comparison with $\eta_1 \propto \eta_2 \propto \sqrt{T_{sc} - \bar{T}}$. Substitution Eq.(20) to the Eqs. (14) and (15) gives the equations of the third order in respect to the amplitudes η_1, η_2 . Analytic solution of this system is possible only at negligibly

small coefficient $|\gamma_2|$. This case at $\gamma_1 > 0$ we obtain

$$\eta_2^2 \cong -\frac{\alpha_{10}}{2\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2}} \left(T - T_{sc0} - \frac{\gamma_1 M_0}{\alpha_{10}} \right), \quad (21)$$

$$\eta_1 \cong \frac{|\gamma_2|}{\alpha_1 + \gamma_1 M_0} \eta_2. \quad (22)$$

This description of the second order phase transition from ferromagnetic to the ferromagnetic superconducting state is valid at the assumption $m \ll M_0$. However, at pressure enhancement the Curie temperature and the critical temperature of superconducting transition (see Fig.1) approach each other, the value of M_0 gets smaller and according to Eq.(20) the value of m increases. One can expect the turning of the second order transition into the first order transition such that the order parameters η_1, η_2, m undergo finite jumps from zero to the finite values at temperature larger than the critical temperature given by Eq.(19). Indeed, the this type of behavior was established in Ref.9 by the numerical solution of nonlinear equations for the order parameter components at close enough values T_{Curie} and T_{sc} .

At tiny $|\gamma_2|$ one can confirm this result analytically making use the standard theory of the first order type transitions near the tricritical point [16]. Indeed, the substitution Eq.(20) to Eq.(13) yields the following term of the fourth order

$$\left(\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2} \right) \eta_2^4. \quad (23)$$

The difference $\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2}$ at small but finite magnitude $M_0(T)$ changes the sign from positive to negative that leads to the first order transition from the normal ferromagnetic state to the superconducting ferromagnetic state.

To establish the whole phase diagram one must consider the phase transition from the normal nonmagnetic state to the superconducting state.

C. Phase transition from the normal metallic to the superconducting state

The case of particular interest is the direct phase transition from the normal state to the superconducting ferromagnetic state which is of the first order according to the results of Ref.9. This case the free energy density (13) minimisation in respect η_1, η_2, M yields

$$\alpha_1 \eta_1 + \gamma_1 M \eta_1 - |\gamma_2| \eta_2 + 2\beta_1 \eta_1^3 + \beta_2 \eta_1 \eta_2^2 = 0, \quad (24)$$

$$\alpha_1 \eta_2 - \gamma_1 M \eta_2 - |\gamma_2| \eta_1 + 2\beta_1 \eta_2^3 + \beta_2 \eta_1^2 \eta_2 = 0, \quad (25)$$

$$2\alpha M + 4\beta M^3 + \gamma_1 (\eta_1^2 - \eta_2^2) = 0. \quad (26)$$

The solution of the last equation has the form

$$M = F(\eta_1^2 - \eta_2^2), \quad (27)$$

where $F(x) = -F(-x)$ is the odd function of its argument. Substituting this formula to Eqs. (24) and (25) we obtain two equations for η_1 and η_2 . These equations are transformed to each other by the interchange of indices $1 \longleftrightarrow 2$. One can say that the first of them determines η_1 as a function of η_2 : $\eta_1 = f(\eta_2)$. And the second equation determines η_2 as the same function of η_1 : $\eta_2 = f(\eta_1)$. This is only possible then

$$\eta_1 = \eta_2. \quad (28)$$

This conclusion is easily checked by the direct calculation. Hence,

$$M = 0. \quad (29)$$

Thus, the superconducting state emerging as result of transition is always nonmagnetic. Another words, **a direct phase transition from the normal paramagnetic to the ferromagnetic superconducting state is impossible.**

The common magnitude of the superconducting amplitudes is

$$\eta^2 = -\frac{\alpha_1 - |\gamma_2|}{2\beta_1 + \beta_2}. \quad (30)$$

At positive sum $2\beta_1 + \beta_2 > 0$ this phase transition is of the second order and occurs at

$$T_{sc} = T_{sco} + \frac{|\gamma_2|}{\alpha_{10}}, \quad (31)$$

that coincides with Eq.(19) at $M_0 = 0$. At $2\beta_1 + \beta_2 < 0$ one can expect the transition of the first order but this situation looks as quite nonrealistic.

So, to pass in the ferromagnetic superconducting state the system must undergo one more phase transition.

D. Phase transition from the superconducting state to the superconducting ferromagnetic state

At this transition the magnetization M spontaneously appears and the superconducting order parameter amplitudes acquire the deviations from the values found in the previous section

$$\eta_1 = \eta + \delta_1, \quad \eta_2 = \eta + \delta_2. \quad (32)$$

The free energy acquires the following form

$$F = \alpha M^2 + \beta M^4 + \alpha_1(\delta_1^2 + \delta_2^2) + \gamma_1 M [2\eta(\delta_1 - \delta_2) + \delta_1^2 - \delta_2^2] - 2|\gamma_2|\delta_1\delta_2 + \beta_1 [6\eta^2(\delta_1^2 + \delta_2^2) + 4\eta(\delta_1^3 + \delta_2^3) + \delta_1^4 + \delta_2^4] + \beta_2 [\eta^2(\delta_1^2 + \delta_2^2 + 4\delta_1\delta_2) + 2\eta\delta_1\delta_2(\delta_1 + \delta_2) + \delta_1^2\delta_2^2] \quad (33)$$

Here we have taken into account that η is the minimum of free energy at $M = \delta_1 = \delta_2 = 0$ and omitted the zero order terms in respect of M, δ_1, δ_2 . The order parameters are determined from the conditions of the free energy minimum

$$\frac{\partial F}{\partial \delta_1} = 0, \quad \frac{\partial F}{\partial \delta_2} = 0, \quad \frac{\partial F}{\partial M} = 0. \quad (34)$$

One can easily check that in linear approximation the equations for $(\delta_1 - \delta_2)$ and M

$$[\alpha_1 + |\gamma_2| + (6\beta_1 - \beta_2)\eta^2](\delta_1 - \delta_2) + 2\gamma_1\eta M = 0, \\ 2\gamma_1\eta(\delta_1 - \delta_2) + \alpha M = 0 \quad (35)$$

are decoupled from the equation for $(\delta_1 + \delta_2)$. Hence, the latter combination is of the next order of smallness in comparison with

$$M \propto (\delta_1 - \delta_2) \propto \sqrt{T_{scM} - T}. \quad (36)$$

Here T_{scM} is the critical temperature of transition from the superconducting to the superconducting ferromagnetic state which is determined from the equation given by the equality to zero of the determinant of the system (35)

$$[\alpha_1 + |\gamma_2| + (6\beta_1 - \beta_2)\eta^2] \alpha - [2\gamma_1\eta]^2 = 0. \quad (37)$$

IV. PHASE DIAGRAM

We have demonstrated that the direct phase transition from the normal nonmagnetic state to the ferromagnetic superconducting state is inadmissible, at least in frame of the model under consideration. Hence, the simple phase diagram with intersection of ferromagnetic and superconducting phase transition lines like it is shown in Fig.1 cannot be realized. One possibility to escape the lines intersection is shown in Fig.2, where the ferromagnetic and the nonmagnetic superconducting state divided by the line of the first order transition. The temperature interval where this transition takes place can be quite small. On the other hand, as we have demonstrated in the section IIIB., to the left of this line in the small pressure interval the transition from the ferromagnetic to the ferromagnetic superconducting state is of the first order. The latter seems to be in correspondence with the sharp drop of resistivity at superconducting phase transition in this pressure interval found in the paper [8].

V. MAGNETOSTATICS

The authors of Ref.9 discussed the phase transition between the SC state and FM+SC state in the neutral Fermi liquid. According to the Section IIID calculations this is the transition of the second order. The situation is changed in a charged Fermi liquid because the magnetization in the superconducting state is inevitably accompanied by the screening currents. The arising of superconducting state in the ferromagnet takes place at finite magnetization. Whereas the arising of ferromagnetic state in the superconductor accompanied by the smooth increasing of magnetization from zero to the finite value. This determines the difference between the transition from the ferromagnetic to the superconducting ferromagnetic state and the transition from the superconducting to the superconducting ferromagnetic state which we discuss here.

Let us consider a cylindrical sample of radius R with axis parallel to the easy magnetization axis. A phase transition to the superconducting ferromagnetic state is accompanied by the appearance of super-currents. The corresponding London equation for magnetic induction is

$$\text{curl} \mathbf{B} = \frac{4\pi \mathbf{j}}{c} = -\frac{\mathbf{A}}{\delta^2} + 4\pi \text{curl} \mathbf{M}, \quad (38)$$

where δ is the London penetration depth. The contribution to the current due to the term $c \text{curl} \mathbf{M}$ is non-vanishing only in the surface layer with thickness of the order of coherence length $\xi \ll \delta \ll R$ [10], hence, the small enough magnetic field has to decay in the sample volume:

$$\mathbf{B}(r) = \mathbf{B}(R) \exp\left(-\frac{R-r}{\delta}\right), \quad (39)$$

where the surface magnetic field is determined by the magnetic moment created by the super-currents flowing in the surface layer

$$\mathbf{B}(R) = 4\pi \mathbf{M}. \quad (40)$$

In UCoGe the ferromagnetic superconducting state arises either from the normal ferromagnetic state or from the nonmagnetic superconducting state.

A. Magnetostatics below transition from the ferromagnetic to the ferromagnetic superconducting state

Just below the temperature of the phase transition from the ferromagnetic to the ferromagnetic supercon-

ducting state discussed in the section **III.B** the field at surface is

$$\mathbf{B}(R) = 4\pi \left(M_0 - \frac{\gamma_1(\eta_1^2 - \eta_2^2)}{8\beta M_0^2} \right). \quad (41)$$

According to the experimental results reported in [17–19] this field at ambient pressure is larger than the lower critical field H_{c1} in UCoGe. So, the complete field screening is not realised, and the phase transition occurs directly to the superconducting mixed state. The vortex cores occupy the small part of the sample volume, and almost whole volume is in the superconducting state with the order parameter given by Eqs.(21), (22). The specific heat jump at phase transition to the superconducting state at the ambient pressure has the finite value

$$\Delta C \cong \frac{\alpha_{10} T_{sc}}{2\beta_1 - \frac{\gamma_1^2}{8\beta M_0^2}}. \quad (42)$$

Here, we have neglected the temperature dependence of $M_0(T)$, just taking it as the constant $M_0 = M_0(T_{sc})$. The specific heat jump at this transition has been registered [4].

B. Magnetostatics below transition from the superconducting to the ferromagnetic superconducting state

Another situation takes place at the transition from the superconducting to the ferromagnetic superconducting state. This case discussed in the section **III.D** the surface field is determined by the small magnetic moment arising at transition in magnetic superconducting state

$$\mathbf{B}(R) \cong 4\pi M \propto \sqrt{T_{scM} - T}. \quad (43)$$

This field is certainly smaller than the lower critical field in well developed superconducting state with superconducting density $n_s \propto \eta^2 \propto (T_{sc} - T)$

$$H_{c1} = 2\pi\mu_b n_s \ln \frac{\delta}{\xi} \propto (T_{sc} - T). \quad (44)$$

The transition to the ferromagnetic superconducting state is characterized by the emergency of magnetic moment M and the magnetic part of superconducting ordering $\sim (\delta_1 - \delta_2)$. But the super-currents completely screen this magnetism in the bulk of material. Thus, the

smooth increase of magnetization from zero to some finite value is not accompanied by a phase transition as it is in the process of the Meissner state formation in a superconductor of the second kind under external magnetic field smaller than H_{c1} .

The pressure decrease stimulates ferromagnetism. Hence, at low temperatures and low pressures the magnetization will exceed the lower critical field and a sample passes to the ferromagnetic superconducting mixed state. So, instead a phase transition between the superconducting and the ferromagnetic superconducting state one can expect just the transition between the Meissner and the mixed superconducting states.

So, the situation is as there is no bulk phase transition at all. The appearance of spontaneous magnetization screened in bulk by the surface super-currents is shown in Fig.2 by the dashed line.

VI. CONCLUSION

The Landau theory allows to establish specific properties of phase transformations in anisotropic ferromagnetic superconducting material UCoGe. There was found that the phase transition from the ferromagnetic to the ferromagnetic superconducting state at ambient pressure is characterized by the appearance of superconducting part of the order parameter whereas the ferromagnetic component does undergo insignificant changes. However, at higher pressures this transition can turn to the transition of the first order.

There was proven that the direct phase transition from the nonmagnetic normal state to the ferromagnetic superconducting state does not take place. Due to this reason the simple intersection of the Curie and the superconducting critical temperature lines is prohibited. As result, the phase diagram acquires the peculiar shape shown in Fig.2.

The magnetic moment at phase transition from the ferromagnetic to the ferromagnetic superconducting state is just partially screened by the superconducting currents. Whereas at phase transition from the superconducting state to the ferromagnetic superconducting state this screening is complete what shades the manifestations of a bulk phase transition. The ferromagnetic mixed superconducting state occurs only at lower pressures where the spontaneous magnetic moment exceeds the lower critical field.

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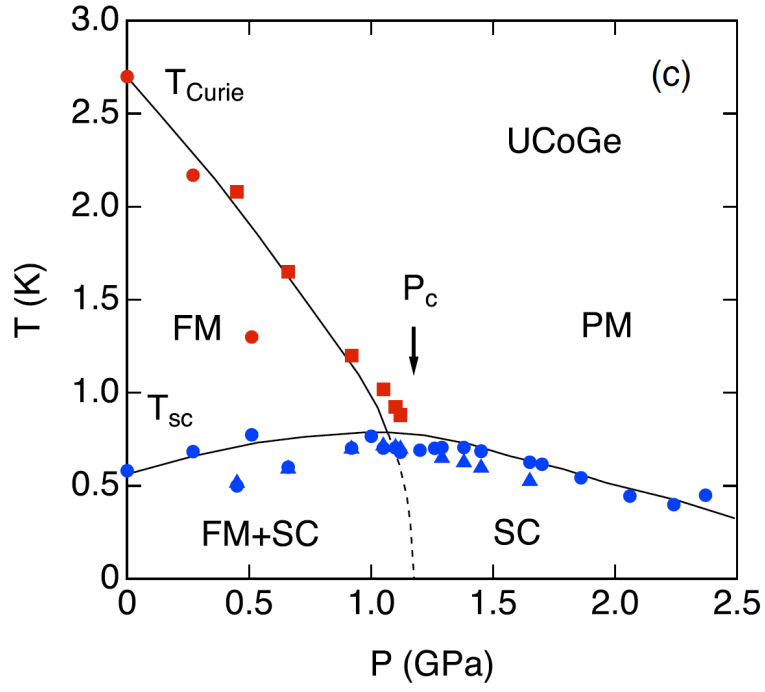


FIG. 1: (Color online) Temperature-pressure phase diagram of UCoGe. Notations FM, SC and PM used for ferromagnetic, superconducting and paramagnetic phases correspondingly [4].

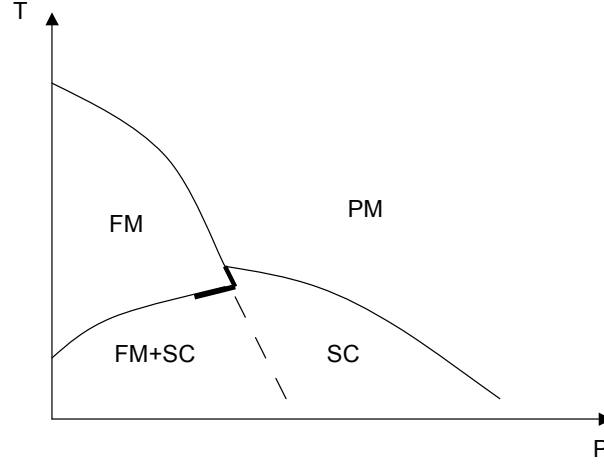


FIG. 2: Schematic temperature-pressure phase diagram of UCoGe. Notations FM, SC and PM used for ferromagnetic, superconducting and paramagnetic phases correspondingly. The thin and thick lines are the lines of the second and the first order transitions correspondingly. The dashed line is the imaginary line of the transition between the nonmagnetic and ferromagnetic superconducting states which is not realized as a phase transition in the bulk of sample.